

Lagrange Interpolation



Lagrange Interpolation

- **1st-order Lagrange polynomial**

$$f_1(x) = L_0 f(x_0) + L_1(x) f(x_1) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

- **Second-order Lagrange polynomial**

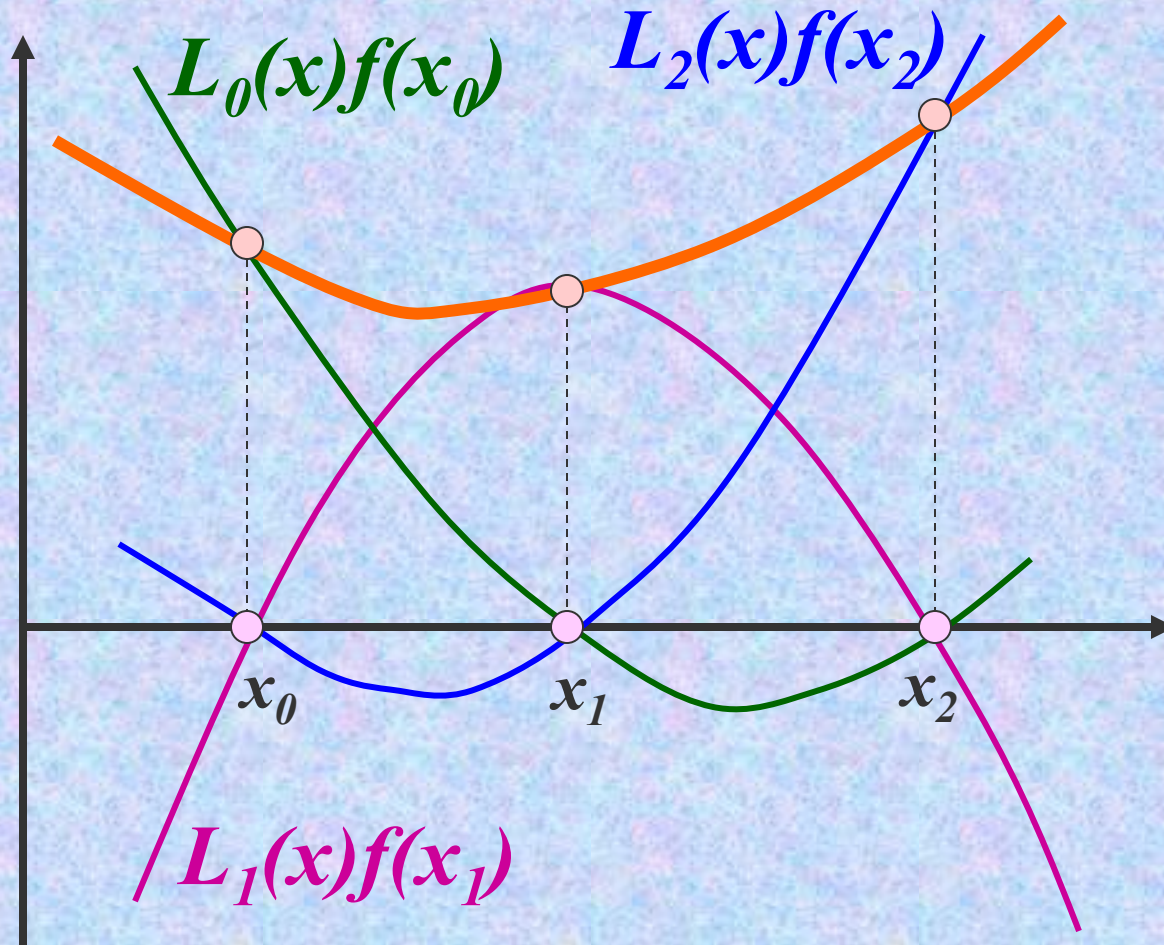
$$\begin{aligned} f_2(x) = & \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} f(x_0) \\ & + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} f(x_1) \\ & + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2) \end{aligned}$$

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- **Third-order Lagrange polynomial**

$$\begin{aligned} f_3(x) = & \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} f(x_0) \\ & + \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} f(x_1) \\ & + \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} f(x_2) \\ & + \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} f(x_3) \end{aligned}$$

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General Three-Point Formula

- Lagrange interpolation polynomial for unequally spaced data

$$\begin{aligned} f(x) &= L_{i-1}(x)f(x_{i-1}) + L_i(x)f(x_i) + L_{i+1}(x)f(x_{i+1}) \\ &= f(x_{i-1}) \frac{(x-x_i)(x-x_{i+1})}{(x_{i-1}-x_i)(x_{i-1}-x_{i+1})} + f(x_i) \frac{(x-x_{i-1})(x-x_{i+1})}{(x_i-x_{i-1})(x_i-x_{i+1})} \\ &\quad + f(x_{i+1}) \frac{(x-x_{i-1})(x-x_i)}{(x_{i+1}-x_{i-1})(x_{i+1}-x_i)} \end{aligned}$$

- First derivative

$$\begin{aligned} f'(x) &= f(x_{i-1}) \frac{2x - x_i - x_{i+1}}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} + f(x_i) \frac{2x - x_{i-1} - x_{i+1}}{(x_i - x_{i-1})(x_i - x_{i+1})} \\ &\quad + f(x_{i+1}) \frac{2x - x_{i-1} - x_i}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)} \end{aligned}$$

Second Derivative

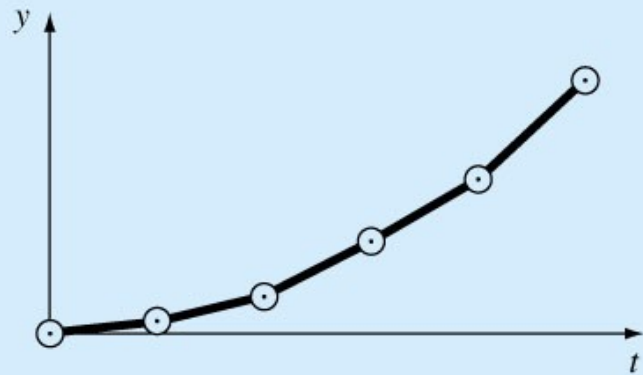
- **First Derivative for unequally spaced data**

$$f'(x) = f(x_{i-1}) \frac{2x - x_i - x_{i+1}}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} + f(x_i) \frac{2x - x_{i-1} - x_{i+1}}{(x_i - x_{i-1})(x_i - x_{i+1})} + f(x_{i+1}) \frac{2x - x_{i-1} - x_i}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)}$$

- **Second Derivative for unequally spaced data**

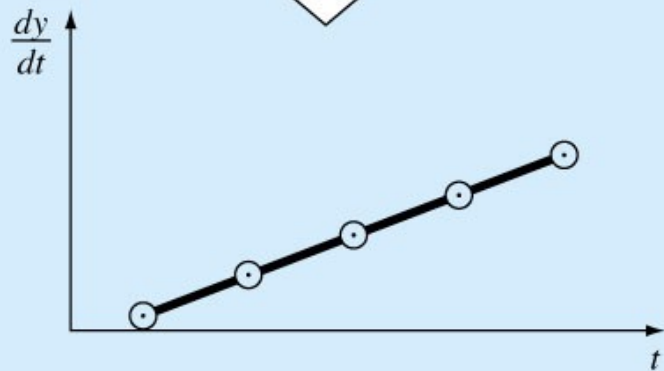
$$f''(x) = f(x_{i-1}) \frac{2}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} + f(x_i) \frac{2}{(x_i - x_{i-1})(x_i - x_{i+1})} + f(x_{i+1}) \frac{2}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)}$$

Differentiation of Noisy Data

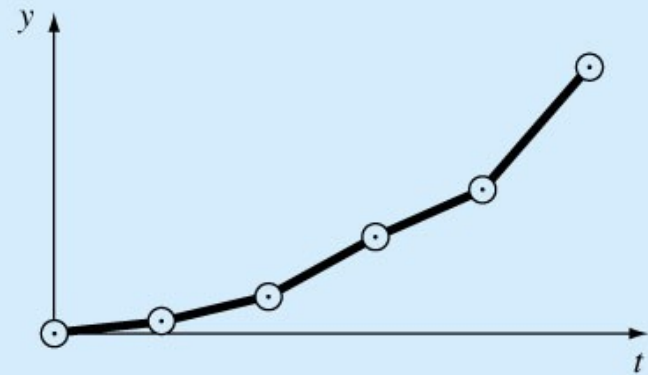


(a)

Differentiate



(c)



(b)

Differentiate



(d)